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## Expectation values in the pilot wave interpretation of quantum mechanics

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**Abstract.** It is shown that the expectation values of position, linear and angular momentum, and potential and total energy, are the same in the pilot wave and usual interpretations of quantum mechanics, but are different for kinetic energy and the square of the linear and angular momentum. Averaging the potential energy over an ensemble in the pilot wave interpretation is shown to be mathematically equivalent to treating  $e|\Psi|^2$  as a charge distribution as in the usual interpretation.

If we substitute  $\Psi = R \exp(iS/\hbar)$ , where  $R$  and  $S$  are real, in Schrödinger's time-dependent equation we obtain, upon separating the real and imaginary parts,

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} = 0, \quad (1)$$

$$\partial R^2 / \partial t + \nabla(R^2 \nabla S / m) = 0. \quad (2)$$

In the pilot wave (or double solution or deterministic) interpretation of quantum mechanics (de Broglie 1927, 1964, Bohm 1952), equation (1) is regarded as a generalisation of the Hamilton–Jacobi equation of motion with  $-\hbar^2(\nabla^2 R)/(2mR)$  interpreted as a quantum potential. Hence the energy  $E = -\partial S/\partial t$ , the momentum  $\mathbf{p} = \nabla S$ , and the velocity  $\mathbf{v} = \nabla S/m$ . From the continuity equation (2), it follows that if  $|\Psi|^2$  initially equals the probability density of position of an ensemble of particles, it will continue to do so at all future times. Bohm (1953) showed for a particular case that, whatever the initial values, the probability density of an ensemble will tend to  $|\Psi|^2$  with time because of collisions, and conjectured that this result was true in general. However, we claim to have shown that it is experimentally feasible to construct ensembles whose probability density does not equal  $|\Psi|^2$  (Robinson 1978, 1980).

We now consider the problem of the expectation values of the dynamical variables: position, linear and angular momentum, and potential and total energy. Writing

$$S = \hbar \tan^{-1} [i(\Psi^* - \Psi)/(\Psi^* + \Psi)],$$

we find that

$$\mathbf{p} = \nabla S = |\Psi|^{-2} \text{Re}(\Psi^* \mathbf{p}_0 \Psi)$$

where the operator  $\mathbf{p}_0 = -i\hbar \nabla$ , as in the usual interpretation of quantum mechanics. Hence, if  $|\Psi|^2$  equals the probability density of position, the expectation value of the

momentum is

$$\langle p \rangle = \int p |\Psi|^2 dv = \text{Re} \int \Psi^* p_0 \Psi dv = \int \Psi^* p_0 \Psi dv.$$

Similarly,

$$E = -\partial S / \partial t = |\Psi|^{-2} \text{Re}(i\hbar \Psi^* \partial \Psi / \partial t) = |\Psi|^{-2} \text{Re}(\Psi^* H_0 \Psi)$$

where  $H_0$  is the usual Hamiltonian operator. Hence,

$$\langle E \rangle = \int E |\Psi|^2 dv = \text{Re} \int \Psi^* H_0 \Psi dv = \int \Psi^* H_0 \Psi dv.$$

A similar identity of the expectation values in the pilot wave and usual interpretations is readily proven for position, angular momentum, and functions of position such as potential energy.

However, the predicted expectation values of the squares of the linear and angular momentum,  $\langle p^2 \rangle$  and  $\langle L^2 \rangle$ , in the two interpretations are, in general, completely different. For example, according to the usual interpretation,  $L^2 = l(l+1)\hbar^2$  for a one-electron atom in the state  $|nlm\rangle$ , and also  $\langle p^2 \rangle = -2\mu E_n$ , where  $\mu$  is the reduced mass. On the other hand, according to the pilot wave interpretation,  $\langle L^2 \rangle = m^2 \hbar^2 / \langle \sin^2 \theta \rangle$  and  $\langle p^2 \rangle = m^2 \hbar^2 / \langle r^2 \sin^2 \theta \rangle$ , so that both quantities are equal to zero when  $m = 0$ , i.e. the electron is stationary (in violation of Heisenberg's uncertainty principle).

An immediate consequence is that Heisenberg's uncertainty principle is not valid in its usual form (Andrade e Silva 1967, de Broglie 1969) and it becomes possible, both in theory (Robinson 1969) and in practice (Robinson 1980), to measure  $x$  and  $p_x$  such that  $\Delta x \Delta p_x < \hbar/2$ . It also follows that hydrogen atoms and those of the alkali metals (in the vapour state) have large permanent electric dipole moments. The problem of permanent electric dipoles in quantum mechanics, and the possibility of testing for their existence in atoms, will be discussed in a future article in collaboration with Professor C E Avedo.

Since in the pilot wave interpretation  $E = K + Q + V$ , where  $K$  is the kinetic energy, and  $Q = -(\hbar^2 \nabla^2 R) / (2mR)$  is the quantic energy, it follows that

$$-\frac{\hbar^2}{2m} \int \Psi^* \nabla^2 \Psi dv = \langle K \rangle + \langle Q \rangle$$

and  $\neq \langle K \rangle$ , as in the usual interpretation. If in a one-electron atom,  $\psi_n$  is an eigenfunction of the Hamiltonian, we have

$$E_n = \langle K \rangle + \langle Q \rangle + \langle V \rangle$$

where

$$\langle V \rangle = -\frac{1}{4\pi\epsilon_0} \int \psi_n^* \frac{Ze^2}{r} \psi_n dv.$$

Thus each atom described by  $\psi_n$  will have in general a different potential, kinetic and quantic energy, but nevertheless the same total energy,  $E_n$ . Furthermore, it is possible that  $V > E$ , since  $Q$ , unlike  $K$ , can be negative, thereby avoiding a difficulty of the statistical interpretation (Claverie and Diner 1976). Finally, we note that the calculation of the potential energy, averaged over an ensemble of atoms each having the

same wavefunction, is mathematically equivalent to treating  $e|\psi|^2$  as the charge distribution of the electron as is done in the usual interpretation. However, as pointed out by Claverie and Diner (1976),  $e|\psi|^2$  is treated as a cloud charge only as far as the interaction between two particles is concerned, while the self-interaction of the cloud is neglected. In the pilot wave interpretation, this inconsistency disappears.

The generalisation of the above for more than one particle is straightforward. For simplicity of notation, we shall limit ourselves to the two-electron atom and neglect exchange effects. If  $\psi_n(\mathbf{r}_1, \mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$ , we have

$$\left[ -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) - \frac{e^2}{4\pi\epsilon_0} \left( \frac{Z}{r_1} + \frac{Z}{r_2} - \frac{1}{r_{12}} \right) \right] \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) = E_n \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2).$$

Multiplying on the left by  $\psi_a^*(\mathbf{r}_1)$  and integrating gives us the Hartree equation

$$-\frac{\hbar^2}{2m}\nabla_2^2\psi_b(\mathbf{r}_2) - \frac{Ze^2}{4\pi\epsilon_0 r_2}\psi_b(\mathbf{r}_2) + \frac{e^2}{4\pi\epsilon_0} \left( \int \psi_a^*(\mathbf{r}_1) \frac{1}{r_{12}} \psi_a(\mathbf{r}_1) d\mathbf{v}_1 \right) \psi_b(\mathbf{r}_2) = E_b \psi_b(\mathbf{r}_2)$$

where

$$E_b = E_n - \langle K_1 \rangle - \langle Q_1 \rangle - \langle V_1 \rangle.$$

Once more we see that averaging the potential energy over the ensemble is mathematically equivalent to treating the electrons as though they were 'clouds' of electric charge without self-interaction.

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